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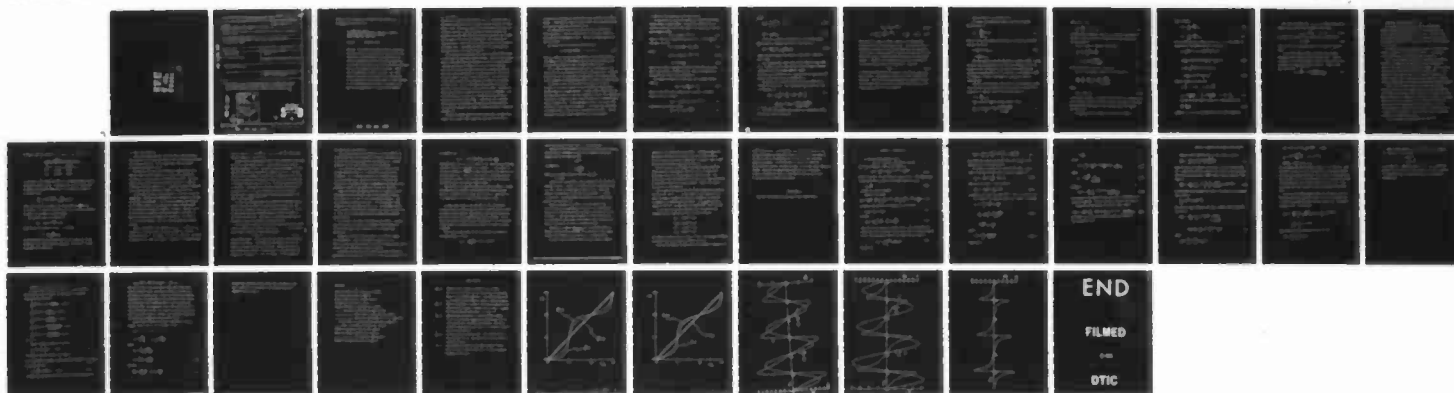
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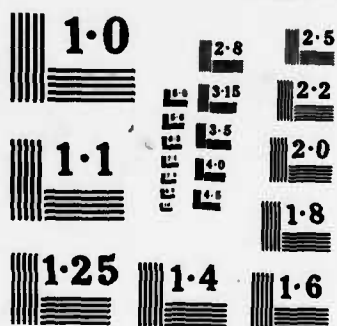
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13. ABSTRACT

Under adiabatic changes of strong magnetic fields, two-dimensional electron systems are expected to show strong temperature oscillations, from which the fundamental constant of the Bohr magneton can be determined. The impurity and temperature sensitivity may have practical applications.

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Effects of level broadening on the magnetothermal oscillations in
two-dimensional electron systems

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Abstract. By a method which modifies Dingle's original one, broadening effects on the magnetothermal oscillations in two-dimensional electron systems are treated analytically and relevant thermodynamic quantities are evaluated explicitly. In particular, the behaviors of the chemical potential and specific heat in a strong magnetic field are investigated. Landau level broadening does not affect the period but causes significant reduction of the amplitude of the oscillations. For its sensitivity, the oscillating pattern is a good indicator of level broadening, while the period of oscillation can be used for the determination of the effective Bohr magneton.

1. Introduction

Two-dimensional electron systems which are formed in Si inversion layers or GaAs/GaAlAs superlattices show very unusual properties especially in strong magnetic field. While these properties have mostly been studied under the isothermal conditions, it has been pointed out (Isihara and Shiwa 1983, Zawadzki and Lassnig 1984) that their temperature oscillates very strongly when the magnetic field is varied adiabatically. The significance of these oscillations is due to the lack of electron motion in the direction of the magnetic field. This makes the conversion of the field energy into the kinetic energy of the electrons effective.

In our previous work, hereafter to be called I (Isihara and Shiwa 1984) we investigated the case without level broadening. Since in actual systems, impurity scatterings cause level broadening, we give in the present article a somewhat comprehensive treatment of the case with level broadening. Different from Zawadzki and Lassnig who employed a numerical approach, we shall make an analytical approach to the magnetothermal effect and derive explicit formulae for relevant physical quantities. For this purpose, we shall employ a method which modifies Dingle's (1952) for the de Haas-van Alphen effect. This is a phenomenological approach, but gives the advantage of being applicable to several different cases. Therefore, we shall treat the cases of Lorentzian as well as elliptic broadenings. For 2D electron systems, the latter type of broadening has been introduced effectively by Ando and Uemura (1984). Although the final broadening effects depend on the magnitude of the respective broadening parameter, it is generally considered that elliptic broadening describes low temperature phenomena well.

In actual 2D systems, there are Coulomb interactions. We remark that Isihara, Tsai and Wadati (1971) showed that these interactions lead to

a reduction of the amplitude of 3D dHvA oscillations similarly to the case of Dingle. More recently, Shiwa and Isihara (1983) treated dHvA oscillations in 2D electron systems with Coulomb interaction.

The magnetothermal effect requires the evaluation of the entropy at finite temperature. This is a difficult task in the presence of impurity scattering and a strong magnetic field. However, the Dingle method can be used effectively.

The energy levels of 2D electrons can be written as

$$\epsilon_n = (2n+1)\mu_B H \pm \frac{g}{2}\mu_B^0 H, \quad n = 0, 1, 2, \dots \quad (1.1)$$

where $\mu_B = e\hbar/2mc$ is the effective Bohr magneton with the effective mass m , $\mu_B^0 = e\hbar/2m_0c$ is the real Bohr magneton with the bare electron mass m_0 , and g is the effective Lande's g factor, H being the magnetic field.

We note that the magnetothermal oscillations are primarily due to the orbital motion of the electrons as in the case of the dHvA effect. Therefore, we shall start with the case without the spin-magnetic field coupling energy. As can be guessed, this coupling causes a shift in the phase of the oscillations.

In the next section, we shall derive a new formula for the grand potential of a 2D electron system with level broadening by a method which modifies slightly the original Dingle's. Section 3 gives a basic formula for the magnetothermal effect for arbitrary broadening and low but finite temperatures. In Section 4, we present an explicit limiting expression for the magnetothermal effect at absolute zero. At the same time, we present a new specific heat formula for the case with level broadening. As we shall see, the behavior of the specific heat is very crucial to the magnetothermal effect. Finally, in Section 5 we present explicit numerical results and discussions.

2. Effects of Level Broadening on the Grand Potential

In this section, we treat level broadening by a method which modifies slightly Dingle's original theory. For analytical simplicity, let us consider an idealized 2D case in which $g\mu_B^0/2$ is set equal to μ_B so that the grand potential becomes

$$\Omega(\mu) = -kT \sum_n \ln[1 + e^{\beta(\mu - \epsilon_n^0)}] \quad (2.1)$$

where $\beta = 1/kT$ is the reciprocal thermal energy and μ is the chemical potential. We write

$$\Omega(\mu) = \frac{1}{2} \Omega_0(\mu + \mu_B H) + \frac{1}{2} \Omega_0(\mu - \mu_B H) \quad (2.2)$$

where

$$\Omega_0(\mu) = -kT \sum_n \ln[1 + e^{\beta(\mu - \epsilon_n^0)}] \quad (2.3)$$

$$\epsilon_n^0 = (2n+1)\mu_B H \quad (2.4)$$

Since the two terms in equation (2.2) differ from each other only in the sign of $\mu_B H$, we derive a formula for $\Omega_0(\mu)$. This is then equivalent to treating the case without the spin contribution.

We introduce the density of states $\phi_0(\epsilon)$ per unit energy interval given by

$$\phi_0(\epsilon) = 2\mu_B H \frac{Am}{\pi \hbar^2} \sum_n \delta(\epsilon - \epsilon_n^0) \quad (2.5)$$

where A is the surface area, m being the electron mass. We express equation (2.3) by an integral

$$\Omega_0(\mu) = -kT \int_0^\infty d\epsilon \phi_0(\epsilon) \ln[1 + e^{\beta(\mu - \epsilon)}] \quad (2.6)$$

Integrating twice by parts, we find

$$\Omega_0(\mu) = \int_0^\infty d\epsilon \phi_2(\epsilon) \left[\frac{\partial f_0(\epsilon)}{\partial \epsilon} \right] \quad (2.7)$$

where

$$\phi_2(\epsilon) = \int_0^\epsilon d\epsilon' \int_0^{\epsilon'} d\epsilon'' \phi_0(\epsilon'') \quad (2.8)$$

$$f_0(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} + 1} \quad (2.9)$$

So far the calculation is formally exact. We now replace Dirac's delta functions by Lorentzian functions of a half width \hbar/τ :

$$\phi_0(\epsilon) = \frac{A\alpha}{\pi\beta} \sum_{n=0}^{\infty} \frac{1}{\pi} \frac{1/\tau}{(\epsilon - \epsilon_n^0)^2 + (1/\tau)^2} \quad (2.10)$$

where, and what follows in this article, we have set $\hbar = 1$ and $2m = 1$, and $\alpha = \beta\mu_B H$.

At low temperatures, $-\partial f_0/\partial\epsilon$ falls off very rapidly on either sides of μ . For $\beta\mu \gg 1$ and $\mu\tau \gg 1$, the n sum may include all negative integers, enabling the use of the Poisson summation formula:

The details of the calculation and a comparison with the original Dingle's method are given respectively in Appendix ^{as} A and B.

The general case in which the electron spins are included can be treated in a similar way. The corresponding grand potential is given for arbitrary g by

$$\Omega(\mu) = -\frac{A}{4\pi} \frac{\eta^2}{\beta^2} \left\{ 1 + \frac{\pi^2}{3\eta^2} + \left[\left(\frac{g}{2} \right)^2 - \frac{D(\Gamma)}{3} \right] \frac{\alpha^2}{\eta^2} \right. \quad (2.11)$$

$$\left. + \frac{4\alpha}{\eta^2} \sum_{l=1}^{\infty} (-1)^{l+1} W^l(\Gamma) \frac{\cos(\pi l/\gamma) \cos(\frac{g}{2}\pi l)}{l \sinh(\pi^2 l/\alpha)} \right\}$$

with $\gamma = \beta/\eta$ and $\gamma = \pi/\eta$.

As we see, the broadening effects appear only through the functions

$D(\Gamma)$ and $W(\Gamma)$ which are defined by

$$W(\Gamma) = \exp[-\pi/(\tau a^2)] \quad (2.12)$$

$$D(\Gamma) = \frac{12}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} e^{-2\pi\Gamma k}; \quad \Gamma = \frac{1}{2\tau a^2}, \quad a^2 = \mu_B H \quad (2.13)$$

Note that $W(\Gamma)$ is simply the Fourier transform of the characteristic Lorentzian form in the sum of equation (2.10). As each energy level is broadened in accordance with this form in energy space, the contribution from the level to the grand potential is reduced by the amplitude reduction factor $W(\Gamma)$ such that the higher harmonic contributions are progressively reduced. Moreover, level broadening affects the non-oscillating part of the grand potential through the function $D(\Gamma)$. Note that

$$D(0) = 1 \quad (2.14)$$

so that the non-oscillating part is reduced to that given by Isihara and Kojima (1979). As broadening increases, that is when Γ increases, $D(\Gamma)$ decreases. This means that the spin paramagnetic contribution becomes relatively more effective, while the oscillating contribution due to the orbital motion of the electrons is weakened.

3. Impurity Effects on Magnetothermal Effect

The temperature changes under adiabatic variations of magnetic field are given by

$$dT = - \frac{\left(\frac{\partial S}{\partial H}\right)_{T,n}}{\left(\frac{\partial S}{\partial T}\right)_{H,n}} dH \quad (3.1)$$

where n is the electron density, and the entropy S is related to the grand partition function by

$$\begin{aligned} S &= \frac{\partial}{\partial T} [\beta^{-1} \ln \Xi]_{H,A,\mu} \\ &= - \left(\frac{\partial}{\partial T} \Omega \right)_{H,A,\mu} \end{aligned} \quad (3.2)$$

The grand potential Ω has been evaluated as in equation (2.29). In order to perform the differentiations of the entropy as in equation (3.1), it is necessary to know the behavior of the relaxation time τ which has been introduced phenomenologically. Let us assume that τ is independent of field and temperature, although it may depend on the electron and impurity concentrations.

Since the electron density is kept constant in the derivatives in equation (3.1), while the grand potential has been obtained for a given chemical potential μ , we need the relation for the number density n :

$$nA = - \left(\frac{\partial \Omega}{\partial \mu} \right)_{T,A,H} \quad (3.3)$$

This yields the important relation between γ and γ_0 :

$$\frac{1}{\gamma_0} = \frac{1}{\gamma} - \frac{2\pi}{\alpha} \sum_{\ell=1}^{\infty} (-)^{\ell+1} w^{\ell}(\Gamma) \frac{\sin\left(\frac{\pi\ell}{\gamma}\right) \cos\left(\frac{g\pi\ell}{2}\right)}{\sinh\left(\frac{\pi^2\ell}{\alpha}\right)} \quad (3.4)$$

where ($M = 1, 2m = 1$)

$$\frac{1}{Y_0} = \frac{2\pi n}{a^2}, \quad \frac{1}{Y} = \frac{\mu}{a^2} \quad (3.5)$$

are dimensionless parameters. Note that $1/Y_0$ represents the experimental variable such as n or H , while $1/Y$ measures the chemical potential by the field energy a^2 .

For $g = 2$ and $T = 0$, equation (3.4) becomes

$$\frac{1}{Y_0} = \frac{1}{Y} + \frac{2}{\pi} \sum_{\ell} \frac{W^{\ell}}{\ell} \sin\left(\frac{\ell\pi}{Y}\right) \quad (3.6)$$

$$= \frac{1}{Y} + \frac{2}{\pi} \tan^{-1} \left[\frac{W \sin\left(\frac{\pi}{Y}\right)}{1 - W \cos\left(\frac{\pi}{Y}\right)} \right] \quad (3.7)$$

The latter formula is valid for $W = \exp(-2\pi\Gamma) < 1$. (3.8)

For $g = 2$, the entropy is obtained in the following form

$$\frac{4\pi S}{Ak} = \frac{2\pi^2}{3\beta} + \frac{4\pi^2}{\beta} \sum_{\ell} W^{\ell} L\left(\frac{\pi^2 \ell}{\alpha}\right) \frac{\cos\left(\frac{\pi \ell}{Y}\right)}{\sinh\left(\frac{\pi^2 \ell}{\alpha}\right)} \quad (3.9)$$

where

$$L(x) = \coth x - \frac{1}{x} \quad (3.10)$$

is the Langevin function. It is worth noting that both $1/Y_0$ of equation (3.4) and S of equation (3.9) do not have the term $D(\Gamma)$ which appears in the grand potential and that the effects of level broadening appear only through the function $W(\Gamma)$.

We compute now the two relevant derivatives of the entropy.

Let us introduce

$$S_H = - \frac{\pi}{Aka^2} H \left(\frac{\partial S}{\partial H} \right)_{T,n} \quad (3.11)$$

$$S_T = \frac{\pi}{Aka^2} T \left(\frac{\partial S}{\partial T} \right)_{H,n} \quad (3.12)$$

After straightforward but somewhat lengthy calculations, which require the derivatives of the chemical potential, we arrive at

$$S_H = \sum_l \frac{\tilde{c}_l}{l} [l'^2 L'(l') - (l' \coth l' + l \tilde{\Gamma}) l' L(l')] - [2\xi \sum_l \tilde{s}_l (l' \coth l' + l \tilde{\Gamma}) + \zeta] [1 + 2\xi l' \tilde{c}_l]^{-1} \cdot \sum_l \tilde{s}_l l' L(l') \quad (3.13)$$

$$S_T = \frac{\xi}{6} + \sum_l \frac{\tilde{c}_l}{l} \{ l'^2 L'(l') - l'^2 (L(l'))^2 \} - 2\xi \left[\sum_l \tilde{s}_l l' L(l') \right]^2 \cdot [1 + 2\xi l' \tilde{c}_l]^{-1} \quad (3.14)$$

where for simplicity we have used

$$\xi = \frac{\pi^2}{\alpha}, \quad \zeta = \frac{\pi}{\gamma}, \quad l' = l\xi = \frac{\pi^2 l}{\alpha} \\ \tilde{c}_l = w^l \frac{\cos(l\xi)}{\sinh l}, \quad \tilde{s}_l = w^l \frac{\sin(l\xi)}{\sinh l}, \quad \tilde{\Gamma} = 2\pi\Gamma = \frac{\pi}{\tau a} \quad (3.15)$$

The magnetothermal effect is expressed in a dimensionless way by

$$\frac{dT}{T} = \frac{S_H}{S_T} \frac{dH}{H} \quad (3.16)$$

Formulae (3.13) and (3.14) are derived for low but finite temperatures.

The formula in equation (3.14) can be used to derive the electronic specific heat at constant area and magnetic field through

$$c_{A,H} = \frac{ka^2}{\pi} S_T \quad (\text{per area}) \quad (3.17)$$

The first term in equation (3.14) yields the linear specific heat

$$c_A^0 = \frac{\pi k^2}{6} T \quad (3.18)$$

which is the correct limiting expression in the absence of a magnetic field.

In an arbitrary magnetic field, the specific heat decreases exponentially with temperature (Shiwa and Isihara 1984). However, so long as Γ , the broadening parameter defined by equation (2.13), does not vanish, the linear specific heat is retained. Indeed, we find near absolute zero (Appendix C)

$$c_{A,H} = \frac{\pi k^2}{6} T \left[1 + 2W \frac{\cos(\pi/\gamma) - W}{1 - 2W \cos(\pi/\gamma) + W} \right] \quad (3.19)$$

4. Magnetothermal Effect at Absolute Zero

The magnetothermal effect as expressed by the dimensionless quantity $HdT/(TdH)$ depends on the two entropy derivatives S_H and S_T given by equations (3.11) and (3.12) respectively. Note that these two quantities are also dimensionless. In the natural units which have been adopted in the present article, the field energy a^2 has the dimension of a reciprocal area.

For absolute zero, the two entropy derivatives can be given explicitly in terms of trigonometric functions as in Appendix C. Our analyses of the case of absolute zero can be summarized as follows:

First, level broadening eases the abrupt variation of the chemical potential as expressed by the dimensionless variable $1/\gamma$ defined by equation (3.5) when $1/\gamma_0$ in the same equation changes. The latter variable represents either the electron density or the magnetic field. This easement is understandable because in the presence of broadening, the chemical potential does not jump from one sharp Landau level to another. Actual broadening effects on the chemical potential must be seen numerically.

Second, level broadening brings back the ideal linear specific heat which vanishes in strong magnetic fields if there is no broadening. Interestingly, this important change in the electronic specific heat takes place irrespective of the magnitude of broadening, if it exists. In a strong magnetic field and in the absence of level broadening, the electronic specific heat varies exponentially with the temperature. Since S_T in the denominator of equation (3.1) is essentially the specific heat, such a change from the exponential to linear variations causes a considerable reduction in the magnetothermal effect.

Let us now examine broadening effects more explicitly.

Two typical numerical values of the ratio S_H/S_T are given in Table 1:

Table 1 Ratio of Entropy Derivatives at 0 K		
$\tilde{\gamma}$	w	$ S_H/S_T $
0.3	0.7408	0.9852
3.0	0.04979	0.2995

In this Table, we have chosen two values of the broadening parameter $\tilde{\gamma}$ which is defined by equation (3.15). Interpreting that $\tau_s = \tau/2 = m\mu/e$ as the scattering time (Brailsford 1966), where μ is the mobility, we find for Si inversion layers

$$\frac{H}{2\tau_s} = 0.3047 \left[\frac{0.19m_0}{m} \right] \left[\frac{10^4}{\mu (\text{cm}^2 \text{V}^{-1} \text{s}^{-1})} \right] \text{ meV}$$

where m_0 and m are the free electron and effective masses respectively.

The notation μ for mobility is used here only; it is the chemical potential everywhere else throughout the present paper.

The cyclotron energy can be expressed as

$$\hbar\omega_c = 0.6093 H \left[\frac{0.19m_0}{m} \right] \text{ meV}$$

where the field H is measured in Tesla. We find then

$$\tilde{\gamma} = \frac{\pi}{H} \left[\frac{10^4}{\mu (\text{cm}^2 \text{V}^{-1} \text{s}^{-1})} \right]$$

We have estimated from this expression that $\tilde{\gamma}$ can be around 0.3. For 3D, McCombe and Adel (1967) adopted the same value. The case of $\tilde{\gamma} = 3$ has been given only for comparison.

5. Results and Discussions

In our grand ensemble approach, the two relevant entropy derivatives with respect to H and T require the evaluation of the corresponding derivatives of the chemical potential. For low temperatures, these are obtained by differentiating equation (3.4).

The important ratio S_H/S_T which determines the magnetothermal effect depends on how the chemical potential μ is related to the experimental variables such as electron density n and the magnetic field H . Note that equation (3.4) represents the chemical potential through $\gamma^{-1} = \mu/\mu_B H$ as a function of the ratio $2\pi n/\mu_B H = \gamma_0^{-1}$. Since their relation is fundamental in our theory, we have illustrated in figure 1 the case with $\tilde{\gamma} = 0.3$.

In this graph, the three curves correspond to $\xi = \pi^2/\alpha = 0, 1.5$ and 5.0 respectively. The first curve represents the case of absolute zero at which the variation is the sharpest. If there is no broadening, the variation at 0 K becomes zig-zag as in our previous work (Shiwa and Isihara 1983). As the temperature increases, the parameter ξ increases. The case $\xi = 1.5$ can be considered intermediate, corresponding roughly to 1.0 K and 2 T. Such an intermediate case is important for experiments. The curve for $\xi = 5.0$ is almost straight, indicating a high temperature and low field relation.

Figure 2 illustrates in the intermediate case of $\xi = \pi^2/\alpha = 1.5$, how changes in $1/\gamma$ take place about odd integral values⁵ of $1/\gamma_0$ when the broadening parameter $\tilde{\gamma}$ is changed. As we see, the smaller $\tilde{\gamma}$ the sharper the variation of $1/\gamma$ about such points. The case $\tilde{\gamma} = 3$ is almost straight

as in the case of $\xi = 5.0$ in figure 1. That is, both $\tilde{\gamma}$ and ξ play similar roles in bringing $1/\gamma$ close to $1/\gamma_0$.

Figure 3 shows the magnetothermal effect at two temperatures corresponding to $\pi^2/\alpha = 5.0$ (left ordinate) and $= 1.5$ (right ordinate) but at the same broadening parameter $\tilde{\gamma} = 0.3$. The arrows indicate the ordinate to be used. These two curves should be compared with figures 3 and 4 in I. We learn that the effect of broadening is strong in the low temperature case of $\pi^2/\alpha = 1.5$ where the amplitude is reduced nearly 1/100, but is not very significant in the high temperature case of $\pi^2/\alpha = 5.0$. The former corresponds roughly to 1 K and 2T. The region of the abscissa corresponds electron density of order 10^{12} cm^{-2} at 2T. Also, comparing the two curves in figure 3 with each other, we learn that the oscillations are more sinusoidal in the higher temperature ^{or lower magnetic field} case.

In view of the coupling between the temperature and broadening effects, we have illustrated in figure 4 two oscillations for absolute zero. The left ordinate corresponds to the case $\tilde{\gamma} = 0.3$ while the right to $\tilde{\gamma} = 3.0$. We find that the amplitude is reduced roughly by a factor of 1/30 due to the increase in broadening. Associated with the amplitude reduction, the oscillating pattern is also changed in an interesting way.

It is important to observe in the above two figures that the nodes appear close to, especially odd, integral values of $1/\gamma_0$ at low temperatures or small $\tilde{\gamma}$ and that the period stays constant:

$$\Delta(1/\gamma_0) = 2 \quad (5.1)$$

regardless ^{of} broadening. On the other hand, the pattern of the oscillations depends on broadening. The changes about odd integral values of the abscissa are sharper than those at even values. However, note that the phase depends on the g-factor. We have chosen $g = 2$ for theoretical convenience, but the case $g = 0$ is also interesting. We shall comment on

the phase in more detail shortly.

The oscillations become more sinusoidal and the nodes deviate from (odd) integral values when the temperature is high or the field is low, as in the case without broadening which was discussed in I. A single parameter $\alpha = a^2/kT$ plays an important role in this respect.

It is important to remark that the presence of broadening causes the zero temperature limit $\alpha \rightarrow \infty$ different from the case without broadening. We have already pointed out that broadening restores the linear specific heat. Since in the absence of broadening, the specific heat is exponentially small, an experimental determination of the temperature variation of the specific heat becomes very important.

Since the specific heat depends sensitively on broadening, we have investigated the zero temperature limit of the magnetothermal effect. Our results for $\tilde{\gamma} = 0.3$ and $\tilde{\gamma} = 3.0$ are illustrated in figure 4. The arrow indicates the ordinate to be used. In the former case, steep changes occur near odd integral values of $1/\gamma_0$. Its oscillating pattern is significantly distorted from the sinusoidal type which is observed for $\tilde{\gamma} = 3.0$. Hence, by observing such pattern changes, level broadening may be assessed.

As in figure 3, the period of oscillation stays constant. Together with the position of the nodes at low temperature and broadening, the effective Bohr magneton may be determined from the magnetothermal oscillations.

However, for actual 2D electron systems it is necessary to extend our consideration. First, let us examine the choice of $g = 2$. This is a special case which simplifies analytical expressions, but in actual inversion layers, the effective g factor is enhanced.

In general, the spin factor enters the grand partition function as a phase factor, and the arbitrary g case can be generated from the case of $g = 0$

in accordance with

$$\cos \pi / \gamma \rightarrow \frac{1}{2} \left(\cos \pi \left(\frac{1}{\gamma} + \frac{g}{2} \frac{m}{m_0} \right) + \cos \pi \left(\frac{1}{\gamma} - \frac{g}{2} \frac{m}{m_0} \right) \right)$$

On the other hand, the case $g = 0$ is similar to the idealized case except that the phase in terms of the variable $1/\gamma_0$ is shifted. For example, in the case corresponding to ^{figure 4} Fig. 5, the sharp increases will take place for $g = 0$ at even integral $1/\gamma_0$. Other than that, there is no significant change in the oscillating pattern. This is understandable because the oscillations are primarily due to the electron's orbital motion.

The electrons in Si inversion layers have the effective mass $m = 0.19m_0$. The effective Lande's factor g and the effective mass modify the phase factor in a combined form of $g m / 2m_0$. This results in a phase shift of ± 0.3 approximately if g is 3 and the effective mass is $0.19m_0$.

A comment must be given on the replacement of the δ functions by Lorentzian functions in view of the work of Ando and Uemura (1974) in which an elliptic form was adopted. They arrived at this form for the central part of each Landau level by neglecting Landau level couplings. On the other hand, Gerhardt (1976) arrived at a Gaussian form for level broadening. In what follows in the present article, we remark on the case of elliptic broadening.

With a proper normalization, the use of an elliptic form amounts in our present theory to have

$$\phi_0(\epsilon) = \frac{A_0}{\pi \beta_0} \frac{2\tau}{\pi} [1 - \tau^2 (\epsilon - \epsilon_n^0)^2]^{1/2}$$

where $\tilde{\tau}$ is generally different from τ of the Lorentzian case.

Adopting the above expression and following the same procedure as before, we find that the c dependence of $\phi_0(c)$ is not changed at all but the damping factor of the Lorentzian case:

$$2\pi e^{-\tilde{\gamma}k}$$

is replaced by

$$4\pi \frac{J_1(\tilde{\gamma}'k)}{\tilde{\gamma}'k}$$

where J_1 is the Bessel function of the first kind, and

$$\tilde{\gamma}' = \pi/\tau a^2$$

Since $\tilde{\gamma}$ and $\tilde{\gamma}'$ are different, a direct comparison of the two cases is somewhat difficult. However, as we remarked before, the scattering time $\tau_s = \tau/2$. Hence, it is meaningful to choose $\tilde{\tau} = \tau/2$. Then, both elliptic and Lorentzian forms will have the same height. Figure

5 illustrates the magnetothermal oscillations for the elliptic form for $\tilde{\gamma}' = 0.6$ and $\pi^2/\alpha = 1.5$.

While the parameter choice $\pi^2/\alpha = 1.5$ for this elliptic case is the same as the corresponding one in figure 3, it is interesting to observe that the oscillating pattern is similar to the curve for $\tilde{\gamma} = 0.3$ in figure 4. The nodes and period appear as in the Lorentzian case, but the amplitude is much larger in the present elliptic case due to the lack of a long tail in the elliptic broadening function.

Although it is somewhat beyond the scope of the present article to discuss in detail the above two ^{broadening} cases, both cases seem to yield similar results if the respective broadening parameter is chosen suitably.

In order to apply the present theory to actual systems, a question

remains as to the observability of the effect. Although discussions of experimental problems are also beyond the scope of the present article, we try to conclude this article by addressing this question. Note in this respect, three-dimensional cases have been observed.

For this purpose, let us consider a model MOSFET (metal-oxide-field effect transistor), consisting of a 50 Å thick Ti gate, a 4,000 Å thick SiO_2 layer, a 50 Å thick inversion layer and a 500 μm Si layer. The surface areas of all these components are assumed to be the same, because then the area does not affect our following estimate. Hence, let us take a unit area for all the components.

Let us consider the case in which the electron temperature is dropped by a few ten percent due to a field change of a few percent at 0.1 K. Under the adiabatic condition, the energy thus released by the electrons is expected to be transferred into the system. For a typical electron density $n \sim 10^{12} \text{ cm}^{-2}$, the heat energy thus generated is $3 \times 10^{-13} \text{ cal/cm}^2$ about.

On the other hand, the heat capacities of all the components are estimated for 0.1 K based on the T^3 and T laws as follows:

$$C_{\text{Ti}} = 3.9 \times 10^{-12} \text{ cal/K}$$

$$C_{\text{SiO}_2} = 8.1 \times 10^{-15} \text{ cal/K}$$

$$C_{\text{el}} = 1.5 \times 10^{-14} \text{ cal/K}$$

$$C_{\text{Si}} = 6.9 \times 10^{-12} \text{ cal/K}$$

The inelastic scattering time is of order 10^{-9} sec. so that we assume equilibrium established rather quickly under the adiabatic condition.

From the above figures, the heat capacity of the MOSFET is estimated to be

1×10^{-11} cal/K at 0.1K. If the above heat energy is entirely absorbed by the system, the temperature change can be around 0.03 K. Since we have neglected the specific heat of the thermometer and its leads, the actual temperature change can be smaller. On the other hand, by reducing the temperature and increasing the magnetic field and its change, one can expect larger temperature changes. Therefore, the magnetothermal oscillations may be observable. At least, two-dimensional systems are more favorable than the case of bulk as far as the energy conversion is concerned.

Acknowledgement

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Appendix A Evaluation of Ω_0

The Poisson summation formula states:

$$\sum_{n=-\infty}^{\infty} f(n) = \int_{-\infty}^{\infty} ds f(s) + 2 \sum_{k=1}^{\infty} \int_{-\infty}^{\infty} ds f(s) \cos(2\pi ks) \quad (A.1)$$

The series in equation (2.10) is then expressed by

$$\phi_0(\epsilon) = \frac{A}{2\pi^2} \left\{ \pi + 2\pi \sum_{k=1}^{\infty} (-)^k e^{-2\pi\Gamma k} \cos\left(\frac{\pi k \epsilon}{a}\right) \right\} \quad (A.2)$$

where $a^2 = \mu_B H$ and Γ has been defined by equation (2.13). This dimensionless broadening parameter is related to the Dingle temperature T_D by

$$T_D = \frac{\Gamma \omega_c}{k\pi} \quad (A.3)$$

ω_c being the cyclotron frequency.

Equation (2.8) yields

$$\phi_2(\epsilon) = \frac{A}{2\pi} \left[\frac{\epsilon^2}{2} + \frac{2a^4}{\pi^2} \sum_{k=1}^{\infty} \frac{(-)^k}{k^2} e^{-2\pi\Gamma k} + \frac{2a^4}{\pi^2} \sum_{k=1}^{\infty} \frac{(-)^{k+1}}{k^2} e^{-2\pi\Gamma k} \cos\left(\frac{\pi k \epsilon}{a}\right) \right] \quad (A.4)$$

Introducing this expression into equation (2.7) and writing

$$\Omega_0(\mu) = \bar{\Omega}_0(\mu) + \tilde{\Omega}_0(\mu) \quad (A.5)$$

we obtain

$$\bar{\Omega}_0(\mu) = \frac{A}{2\pi} \int_0^{\infty} d\epsilon \left[\frac{\epsilon^2}{2} - \frac{a^4}{6} D(\Gamma) \right] \left(\frac{\partial f_0}{\partial \epsilon} \right) \quad (A.6)$$

where $D(\Gamma)$ has been defined by equation (2.13). Note that the series in equation (A.4) are characterized by the reduction factor W defined by equation (2.12). The functions $D(\Gamma)$ and $W(\Gamma)$ are related with each other by

$$D(\Gamma) = \frac{12}{\pi^2} \int_0^{\infty} \frac{W(\Gamma)}{t} dt \frac{\ln(1+t)}{t} \quad (A.7)$$

We also find

$$\tilde{\Omega}_0(\mu) = \frac{Aa^4}{\pi^3} \sum_{k=1}^{\infty} \frac{(-)^{k+1}}{k^2} e^{-2\pi\Gamma k} \int_0^{\infty} d\epsilon \cos\left(\frac{\pi k \epsilon}{a}\right) \left(\frac{\partial f_0}{\partial \epsilon}\right) \quad (\text{A.8})$$

Note that $\bar{\Omega}_0(\mu)$ represents the non-oscillating part and $\tilde{\Omega}_0(\mu)$ is the oscillating part of the grand potential.

The integral in equation (A.6) can be obtained easily. In the neglect of exponentially small terms, we arrive at (Appendix B)

$$\bar{\Omega}_0(\mu) = -\frac{A}{4\pi} \left[\mu^2 + \frac{\pi^2}{3\beta^2} - \frac{a^4}{3} D(\Gamma) \right]. \quad (\text{A.9})$$

Note that in the limit $\Gamma = 0$ this reduces to

$$\bar{\Omega}_0(\mu) = -\frac{A}{4\pi} \frac{\eta^2}{\beta^2} \left[1 + \frac{\pi^2}{3\eta^2} - \frac{\gamma^2}{3} \right] \quad (\text{A.10})$$

in agreement with Isihara and Kojima (1979). Here, $\eta = \beta\mu$ and $\gamma = \alpha/\eta$.

The integral in the oscillating part can be obtained as follows:

$$\begin{aligned} I_k &= -\beta^{-1} \int_0^{\infty} d\epsilon \frac{\partial f_0}{\partial \epsilon} \cos\left(\frac{\pi k \epsilon}{a}\right) \\ &\approx \frac{2}{\beta} \cos\left(\frac{\pi k}{\gamma}\right) \int_0^{\infty} dx \frac{e^x \cos\left(\frac{\pi k}{\alpha} x\right)}{(e^x + 1)^2} \\ &= \frac{\cos\left(\frac{\pi k}{\gamma}\right)}{\beta} \operatorname{Re} I'\left(\frac{\pi k}{\alpha}\right) \end{aligned} \quad (\text{A.11})$$

where

$$I'(\delta) = \int_{-\infty}^{\infty} dx \frac{\exp(x + i\delta x)}{(e^x + 1)^2} \quad (\text{A.12})$$

Substituting

$$z = \frac{1}{e^x + 1} \quad (\text{A.13})$$

we get

$$\tilde{I}^{\pm}(\delta) = \int_0^1 dz \frac{(1-z)^{1\delta}}{z^{1\delta}} = \frac{\Gamma(1+i\delta)\Gamma(1-i\delta)}{\Gamma(2)} = \frac{\pi\delta}{\sinh(\pi\delta)} \quad (\text{A.13})$$

Hence,

$$I_k = \frac{\cos(\frac{\pi k}{\gamma})}{8\alpha} \frac{\pi^2 k}{\sinh(\frac{\pi^2 k}{\alpha})} \quad (\text{A.15})$$

Therefore,

$$\tilde{\Omega}_0(\mu) = -\frac{\Lambda\alpha}{\pi\beta^2} \sum_{k=1}^{\infty} (-)^{k+1} e^{-2\pi\Gamma k} \frac{\cos(\pi k/\gamma)}{k \sinh(\pi^2 k/\alpha)} \quad (\text{A.16})$$

The appearance of the exponential factor $\exp(-2\pi\Gamma k)$ is characteristic of Lorentzian broadening. Note that this factor decreases rapidly with k .

Adding the two contributions, we arrive at a basic formula:

$$\Omega_0(\mu) = -\frac{\Lambda}{4\pi} \left[\mu^2 + \frac{\pi^2}{3\beta^2} - \frac{D(\Gamma)}{3} a^4 + \frac{4\alpha}{\beta^2} \sum_{\ell=1}^{\infty} (-)^{\ell+1} e^{-2\pi\Gamma\ell} \frac{\cos(\pi\ell/\gamma)}{\ell \sinh(\pi^2\ell/\alpha)} \right] \quad (\text{A.17})$$

Accordingly, we can construct the grand potential for arbitrary g following equation (2.2).

Appendix B Comparison with Dingle's Original Method

(Dingle's original method amounts to replacing equation (2.6) by

$$\Omega_0^D(\mu) = -\frac{\Lambda\alpha}{\pi\beta^2} \sum_{n=0}^{\infty} \frac{\Gamma}{\pi} \int_{-\infty}^{\infty} d\epsilon \frac{\ln[1+e^{\beta\mu-2\alpha\epsilon}]}{[\epsilon-(n+\frac{1}{2})]^2+\Gamma^2} \quad (\text{B.1})$$

where the suffix D is attached for distinction. Note that the lower bound is $-\infty$. Dingle gave a physical ground for this, although not very convincing. The n-sum can be replaced by an integral based on Poisson's sum rule. The result is

$$\Omega_0^D(\mu) = -\frac{\Lambda\alpha}{\pi\beta^2} \frac{\Gamma}{\pi} \sum_{k=-\infty}^{\infty} (-)^k \int_{-\infty}^{\infty} d\epsilon \int_0^{\infty} ds \frac{e^{2\pi i k s}}{(\epsilon-s)^2+\Gamma^2} \ln(1+e^{\beta\mu-2\alpha\epsilon}) \quad (\text{B.2})$$

The s-integral can be performed, but results in divergences in the ϵ -integral such as seen in

$$\int_{-\infty}^{\infty} d\epsilon e^{2\pi i k \epsilon} \ln(1+e^{\beta\mu-2\alpha\epsilon})$$

Neglecting all these divergences, and after a somewhat lengthy calculation, we arrive at

$$\begin{aligned} \Omega_0^D(\mu) = & -\frac{\Lambda\alpha}{\pi\beta^2} \left\{ \sum_{\ell=1}^{\infty} (-)^{\ell+1} e^{-2\pi\ell\Gamma} \frac{\cos(\frac{\pi\ell}{Y})}{\ell \sinh(\frac{\pi^2\ell}{\alpha})} \right. \\ & \left. + 2\alpha[g_2(\frac{1}{2Y}) + \sum_{n=1}^{\infty} \tilde{B}_n \alpha^{-2n} g_0^{[2(n-1)]}(\frac{1}{2Y})] \right\} \quad (\text{B.3}) \end{aligned}$$

where

$$g_2(x) = \int_{-\infty}^x dx' \int_{-\infty}^{x'} dx'' g_0(x'')$$

$$g_0(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{x}{\Gamma}\right) + \sum_{p=1}^{\infty} \tilde{b}_p \left(\frac{\partial}{\partial x}\right)^{2p-1} \frac{1}{\pi} \frac{\Gamma}{x^2 + \Gamma^2}$$

$$\tilde{b}_p = (-)^p \frac{(1-2^{1-2p})B_p}{(2p)!}, \quad \tilde{b}_n = (-\pi^2)^n \tilde{b}_n \quad (B.4)$$

$$g_0^{[v]} \left(\frac{1}{2\gamma} \right) = \left[\left(\frac{\partial}{\partial z} \right)^v g_0(z) \right]_z = 1/2\gamma$$

and B represents the Bernoulli number such as $B_0 = -1$, $B_1 = 1/6$, etc. Note that the first term $1/2$ of $g_0(x)$, when introduced into the double integral for $g_2(x)$, still causes a divergence. We must neglect this divergence also. Then, for the case of $\mu\tau \gg 1$, the nonoscillating part of equation (B.3) yields the same diamagnetic terms as Dingle's (See his equation (4.2)). Also, we can show that (B.3) is reduced to the correct expression in the limit $\Gamma \rightarrow 0$.

In our present somewhat improved method, the replacement of Dirac's δ functions by Lorentzian functions is made after $\Omega_0(\mu)$ is integrated twice by parts as in equation (2.7) where the lower bound of the ϵ integration is kept to be $0+$. We then obtain

$$\begin{aligned} \Omega_0(\mu) = & -\frac{4A\alpha^3}{\pi\beta^2} \int_{0+}^{\infty} d\epsilon \frac{e^{2\alpha\epsilon - \eta}}{(e^{2\alpha\epsilon - \eta} + 1)^2} \left[g_2(\epsilon) + \sum_{k=-\infty}^{\infty} (-)^k e^{-2\pi\Gamma|k|} \right. \\ & \left. \times \int_{0+}^{\epsilon} d\epsilon' \int_{0+}^{\epsilon'} d\epsilon'' e^{2\pi i k \epsilon''} \right] \end{aligned} \quad (B.5)$$

where \sum'_k means summation excluding the term $k = 0$, and

$$g_2(\epsilon) = \int_{0+}^{\epsilon} d\epsilon' \int_{0+}^{\epsilon'} d\epsilon'' g_0(\epsilon'') \quad (B.6)$$

Under the dHvA condition that $\gamma \ll 1$, $g_0(1/2\gamma)$ is a power series in γ^2 . If $1/\gamma \gg \Gamma$, i.e., $\mu\tau \gg 1$ then

$$g_0(\epsilon) \sim 1$$

$$g_2(\epsilon) \sim \epsilon^2/2$$

in the region of ϵ immediately close to $1/\gamma$. Hence, we arrive at equations (A.9) and (A.16). Note in equation (A.9), $D(\Gamma)$ appears in the nonoscillating part, reducing the magnitude of the diamagnetic susceptibility.

Appendix C Treatment for Absolute Zero

In this section, we consider the limits $\alpha \rightarrow \infty$ and $\beta/\tau \rightarrow \infty$ and try to derive explicit limiting formula. For this purpose, we introduce simplifying notations such as

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{a^n}{n} \sin nx &= \tan^{-1} \left[\frac{a \sin x}{1 - a \cos x} \right] = S_{-1}(x; a) \\ \sum_{n=1}^{\infty} a^n \sin nx &= \frac{a \sin x}{1 - 2a \cos x + a^2} = S_0(x; a) \\ \sum_{n=1}^{\infty} a^n \cos nx &= \frac{a \cos x - a^2}{1 - 2a \cos x + a^2} = C_0(x; a) \\ \sum_{n=1}^{\infty} a^n n \cos nx &= \frac{a[(1+a^2) \cos x - 2a]}{(1 - 2a \cos x + a^2)^2} = C_1(x; a) \\ \sum_{n=1}^{\infty} a^n n \sin nx &= \frac{a(1-a^2) \sin x}{(1 - 2a \cos x + a^2)^2} = S_1(x; a) \end{aligned} \tag{C.1}$$

for $|a| < 1$.

Hence, γ versus γ_0 relation (3.7) reads as

$$\frac{1}{\gamma_0} = \frac{1}{\gamma} + \frac{2}{\pi} S_{-1} \left(\frac{\pi}{\gamma}; w \right), \quad (\alpha \rightarrow \infty) \tag{C.2}$$

It is easy to show that equation (3.13) leads to

$$S_H = - \frac{\pi^2}{3\alpha} \{ \tilde{r} C_1 \left(\frac{\pi}{\gamma}; w \right) + S_1 \left(\frac{\pi}{\gamma}; w \right) \left[\frac{\pi}{\gamma} + 2S_{-1} \left(\frac{\pi}{\gamma}; w \right) + 2\tilde{r} S_0 \left(\frac{\pi}{\gamma}; w \right) \right] [1 + 2C_0 \left(\frac{\pi}{\gamma}; w \right)]^{-1} \}, \quad (\alpha \rightarrow \infty, \beta/\tau \rightarrow \infty) \tag{C.3}$$

We can also show that

$$S_T = \frac{\pi^2}{6\alpha} [1 + 2C_0 \left(\frac{\pi}{\gamma}; w \right)], \quad (\alpha \rightarrow \infty) \tag{C.4}$$

We remark that this entropy derivative yields the specific heat as follows:

$$c_{A,H} = \frac{ka^2}{\pi} S_T = \frac{\pi k^2}{6} T \{1 + 2C_0 \left(\frac{\pi}{\gamma}, W\right)\} \quad , \text{ (per area)} \quad (C.5)$$

Note that the first term is the correct linear specific heat in the absence of magnetic field. This term is canceled out if there is no broadening of the Landau levels. In fact, the specific heat decreases exponentially. The reappearance of the linear specific heat is due to the introduction of level broadening which in effect suppresses the oscillating terms. The reappearance of the linear specific heat is in accord with Isihara and Kojima who treated the oscillating terms to be small even without broadening. However, their assumption is valid only for relatively high temperature and low magnetic field.

These limiting formulae can further be simplified if $1/\gamma$ is an integer. If $W \neq 1$ and $1/\gamma = m$, an integer, we find

$$S_n = 0 \quad n = -1, 0, 1$$

$$C_0 = \frac{(-)^m W}{1 - (-)^m W} \quad C_1 = \frac{(-)^m W}{[1 - (-)^m W]^2} \quad (C.6)$$

Hence,

$$S_H = - \frac{\pi^2}{3\alpha} \frac{(-)^m W^{\tilde{m}}}{[1 - (-)^m W]^2} \quad (C.7)$$

$$S_T = \frac{\pi^2}{3\alpha} \left\{ \frac{1}{2} + \frac{(-)^m W}{1 - (-)^m W} \right\} \quad (C.8)$$

We obtain

$$\frac{dT}{T} = -\frac{2W^{\tilde{m}}}{1-W^2} \frac{dH}{H} \quad \text{for } \frac{1}{\gamma} = \begin{cases} \text{even} \\ \text{odd} \end{cases} \quad (C.9)$$

Although these expressions involve series, their convergence is very good in general due to the presence of the convergence factor w^l appearing through \tilde{c}_l and \tilde{s}_l .

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Figure Captions

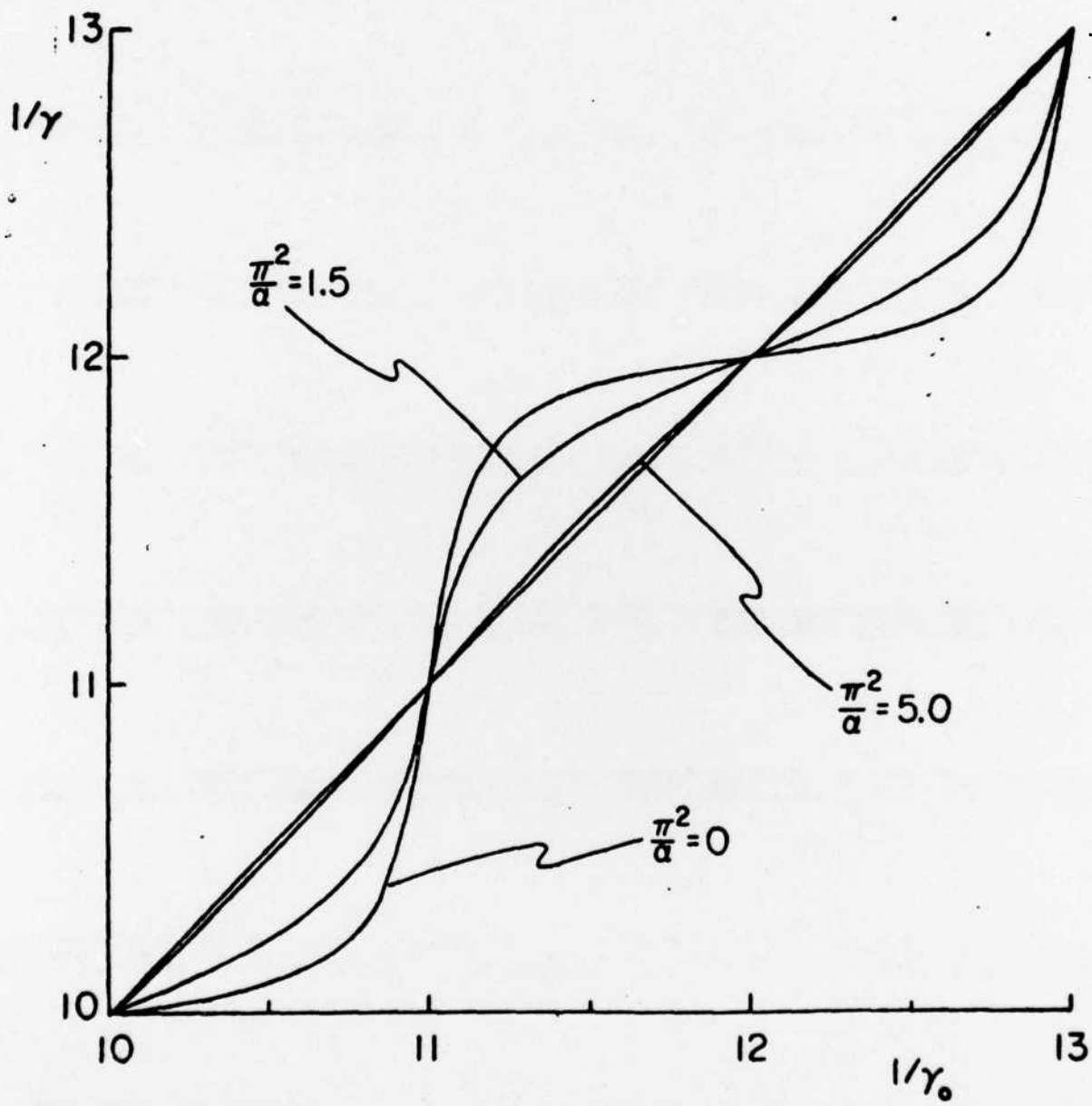
Fig. 1. Variation of $1/\gamma = \mu/\mu_B H$ as a function of $1/\gamma_0 = 2\pi n/\mu_B H$ ($M = 1, 2m = 1$) at $\tilde{\gamma} = 2\pi\Gamma = 0.3$ for three different temperatures. Here, μ is the chemical potential, n is the electron density and μ_B is the effective Bohr magneton, Γ being a broadening parameter.

Fig. 2. Effect of broadening on $1/\gamma$ which is the chemical potential in the units of the field energy at constant $\pi^2/\alpha = 1.5$, where $\alpha = \mu_B H/kT$. This case corresponds approximately to 1 K and 2T.

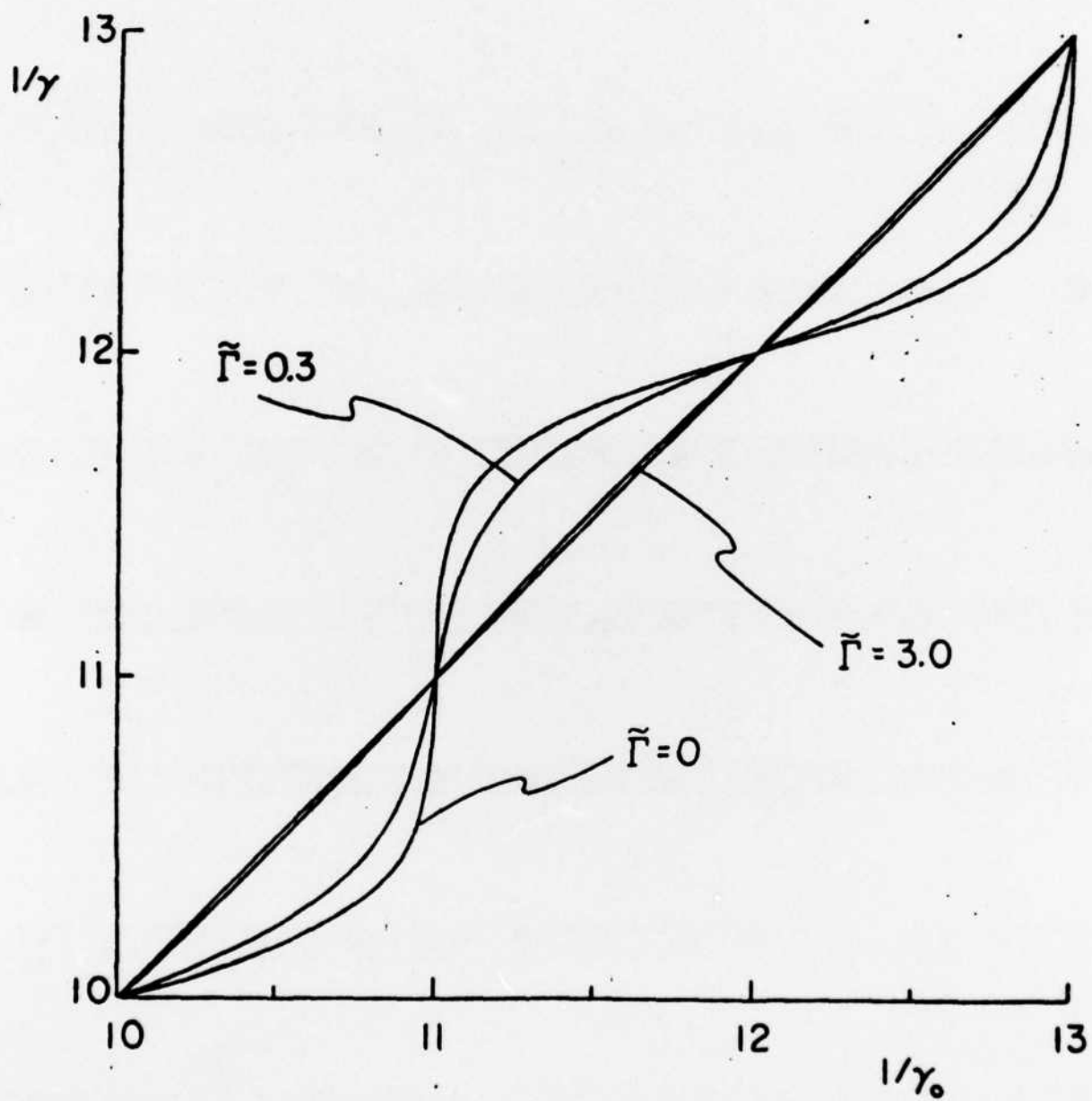
Fig. 3. Magnetothermal oscillations for a fixed broadening parameter $\tilde{\gamma} = 2\pi\Gamma = 0.3$ at different temperatures corresponding to $\pi^2/\alpha = 5.0$ (left ordinate) and $\pi^2/\alpha = 1.5$ (right ordinate). The latter represents roughly the case of 2T and 1K in the region of electron density of order $10^{12}/\text{cm}^2$.

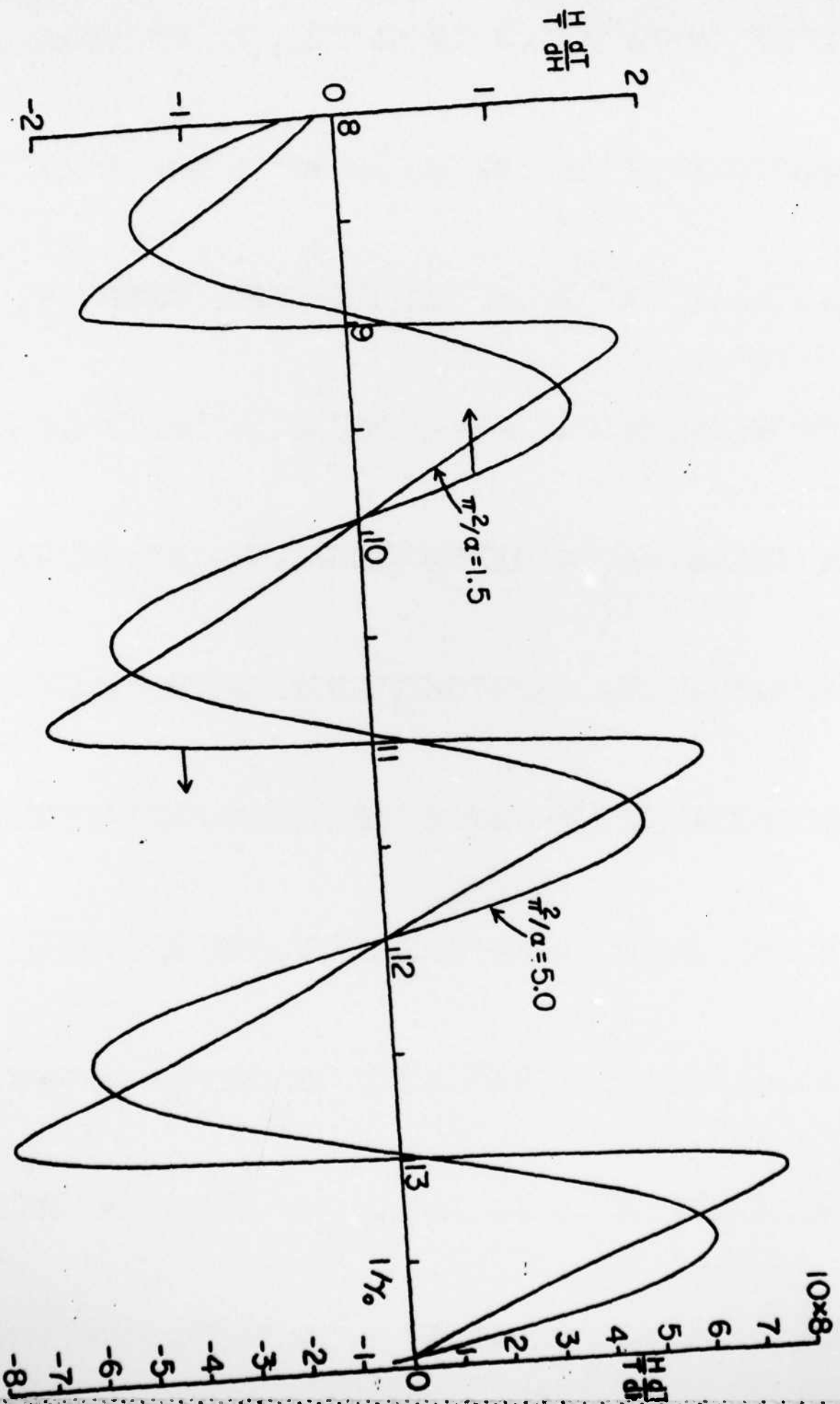
Fig. 4. Broadening effects at absolute zero. Left ordinate: $\tilde{\gamma} = 0.3$. Right ordinate: $\tilde{\gamma} = 3.0$. The arrow in each curve represents the ordinate to be used.

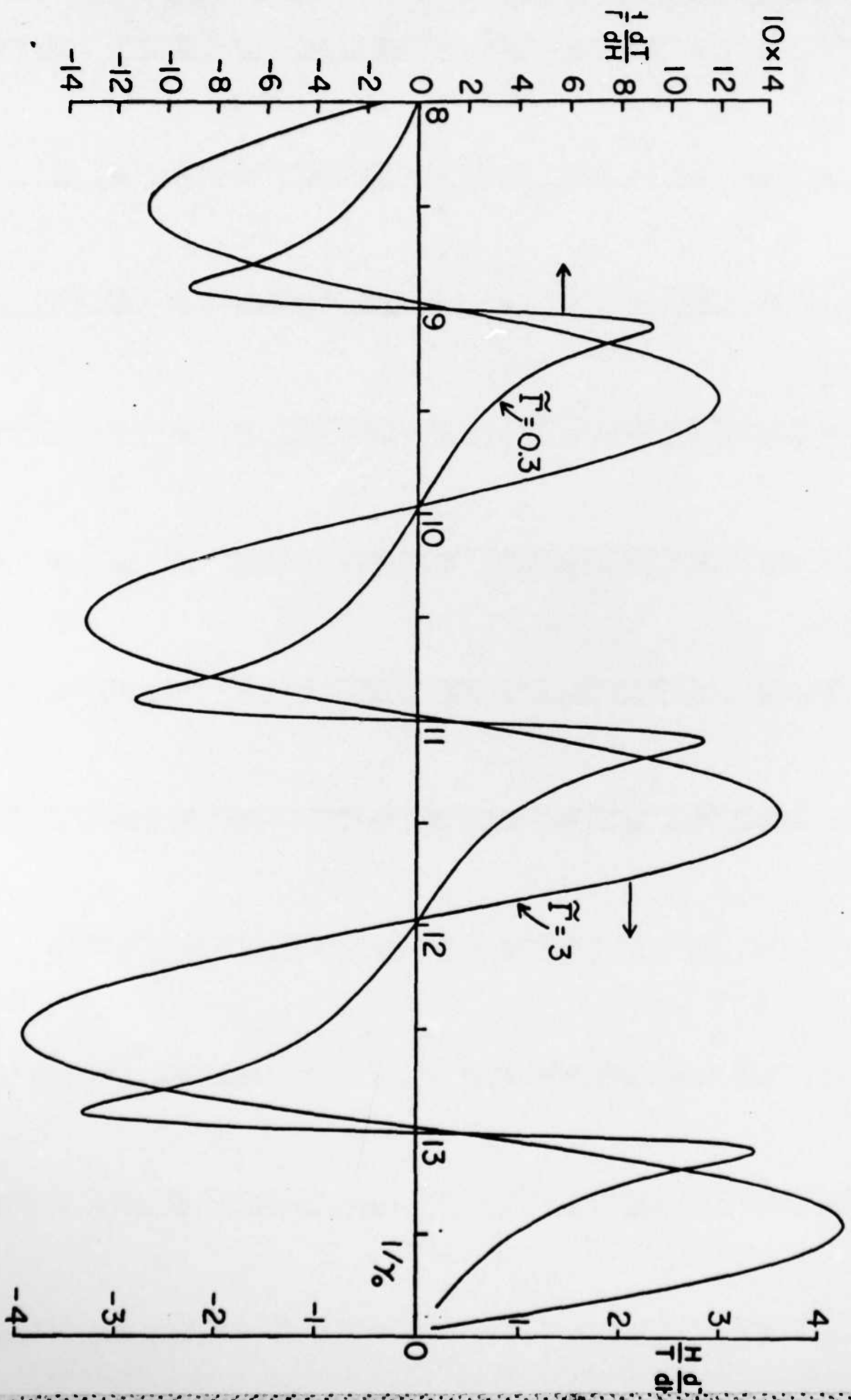
Fig. 5. Magnetothermal oscillations for an elliptic density of states with $\tilde{\gamma}' = 2\pi\Gamma' = 0.6$ and for $\pi^2/\alpha = 1.5$. Γ' is the broadening parameter in this case.

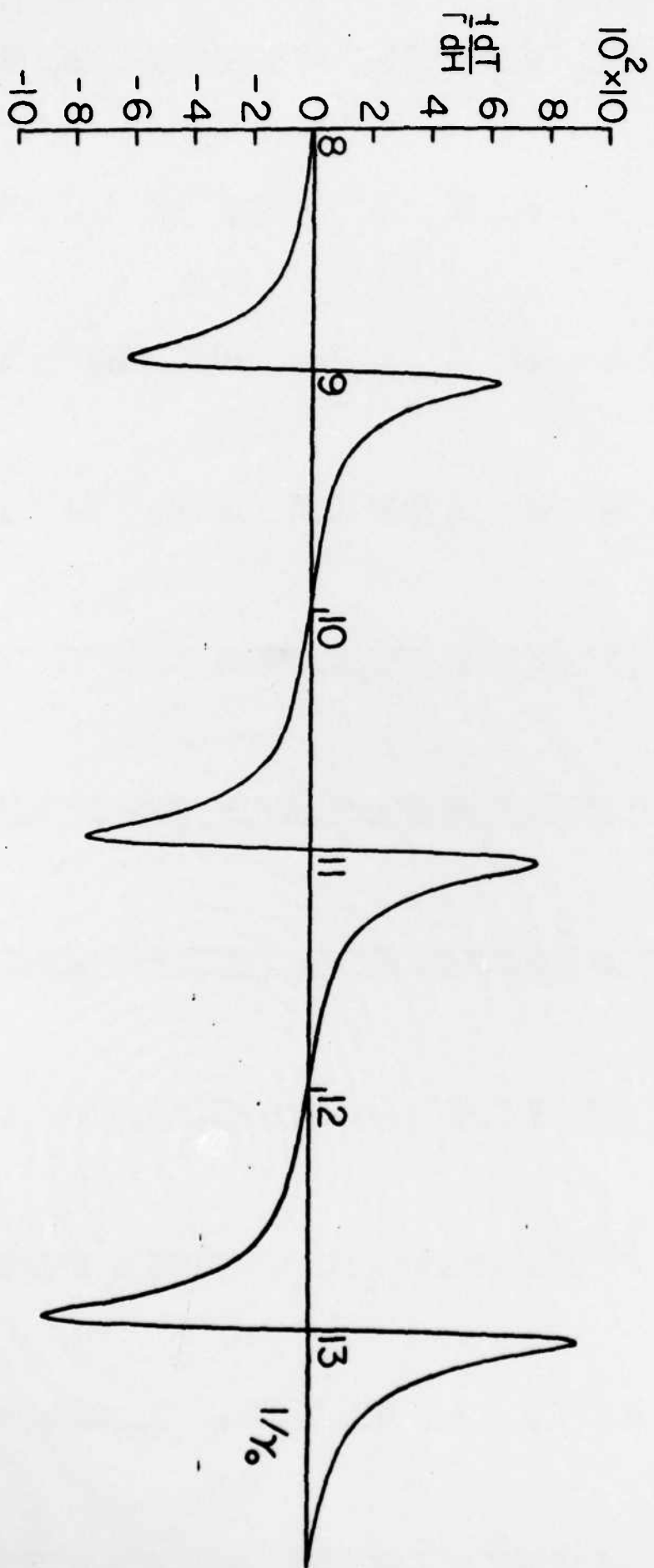


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$$S_T = \frac{1}{6\alpha} [1 + 2C_0(\frac{1}{\gamma}, W)] , \quad (1)$$

We remark that this entropy derivative yields the specific heat as follows:

$$\frac{\overline{H}}{T} = \frac{1}{1-W^2} + \frac{\overline{H}}{H} \quad \text{for } \gamma = 1 \text{ odd}$$

(C.9)

